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Signals & Systems

Problem 1 Solution

1(a) Steady solution : $\frac{dV_0}{dt} = 0$

$$\Rightarrow -g - \frac{1}{2}\rho V_0^2 \frac{C_D A}{m} + \frac{T_0}{m} = 0 \qquad (1)$$

$$\boxed{V_0 = \sqrt{\frac{2m}{\rho C_D A}\left(\frac{T_0}{m} - g\right)}}$$

1(b) Subsitute $V(t) = V_0 + V'(t)$, $T(t) = T_0 + T'(t)$
into the nonlinear equation:

$$\frac{d}{dt}(V_0 + V') = -g - \frac{1}{2}\rho (V_0 + V')^2 \frac{C_D A}{m} + \frac{T_0}{m} + \frac{T'}{m}$$

note $\frac{dV_0}{dt} = 0$, and define $\alpha = \frac{\rho C_D A}{m}$

$$\frac{dV'}{dt} = -g - \frac{1}{2}\alpha(V_0^2 + 2V_0 V' + V'^2) + \frac{T_0}{m} + \frac{T'}{m}$$

From (1) above, we see $-g - \frac{1}{2}\alpha V_0^2 + \frac{T_0}{m} = 0$, so

$$\frac{dV'}{dt} = -\alpha V_0 V' - \frac{\alpha}{2} V'^2 + \frac{T'}{m}$$

To linearize: for small V', we neglect the V'^2 term.

Linearized model:

$$\boxed{\frac{dV'}{dt} = -\alpha V_0 V' + \frac{T'}{m}} \qquad \text{where } \alpha = \frac{\rho C_D A}{m}$$

[Note: could also use Taylor series expansion of

$$f(V) = \frac{1}{2}\alpha V^2$$
$$= f(V_0) + \left.\frac{df}{dV}\right|_{V_0}(V - V_0) + \frac{1}{2}\left.\frac{d^2f}{dV^2}\right|_{V_0}(V - V_0)^2 + \cdots$$
$$= \frac{1}{2}\alpha V_0^2 + \alpha V_0 V' + \frac{1}{2}\alpha V'^2$$

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